BOUNDARY VALUES OF BERGMAN-HARMONIC MAPS

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ABSTRACT. We start from A. Korányi & H.M. Reimann's crucial observation (cf. [4]) that, as a consequence of Fefferman's asymptotic expansion formula (cf. [1]) for the Bergman kernel of a smoothly bounded strictly pseudoconvex domain $\Omega \subset \mathbb{C}^n$

$$K(\zeta, z) = C_\Omega |\nabla \varphi(z)|^2 \cdot \det L_\varphi(z) \cdot \Psi(\zeta, z)^{-(n+1)} + E(\zeta, z),$$

$$|E(\zeta, z)| \leq C'_\Omega \, |\Psi(\zeta, z)|^{-(n+1)+1/2} \cdot |\log |\Psi(\zeta, z)|| \, ,$$

the Kählerian geometry of the interior of Ω may be effectively related to the contact geometry of its boundary $\partial\Omega$. Then we make use of the Graham-Lee connection (cf. [2]) to derive the compatibility equations on $\partial\Omega$ satisfied by the boundary values of a Bergman-harmonic map $\Phi : \overline{\Omega} \to S$ which is C^∞ up the boundary. We are led to a geometric interpretation of Jost & Xu's subelliptic harmonic maps (cf. [3]) from an open set $U \subset \mathbb{R}^{2n-1}$ carrying a given Hörmander system of vector fields.

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